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DETERMINATION OF THERMAL DIFFUSIVITY OF A MAIZE COB

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Key words : Maize cob, Thermal diffusivity, Temperature distribution, Finite difference method

ABSTRACT

This study is concerned with the development of temperature profile inside a maize cob . There was no significant difference in the temperature distribution through the central region during heating or cooling. A faster temperature response was noticed in the germ part. The resistance to heat flow seems to be considerable in a region between the germ and the root of the kernels. Airflow has no significant effect on the temperature distribution , at least within the velocity range of 1.14 m/s to 2.33 m/s. Thermal diffusivities were calculated by graphical method and found to be in close agreement with the results for other cereals.

INTRODUCTION

Through out the world, tonnes of maize cobs are grown each year. Designing of dryers or storage systems for maize cobs on a rational basis requires a knowledge of physical properties of maize cob. To accurately explain the effects of various drying conditions on maize cob, the temperature response and thermal diffusivity of cobs must be known.

Pabis and Hall (1960) investigated the temperature distribution outside and inside an ear of corn placed parallel to the direction of air flow. They noticed that the temperature distribution curves for an ear of corn were considerably different from those which are to be expected when a homogenous body is heated.

Ott and Hurbut (1964) investigated the thermal diffusivity of alfalfa hay to gain a better understanding of temperature distribution in spontaneous heating of stored hay.

Karzarian and Hall (1965) found that the thermal diffusivity of wheat is a non-linear function of moisture content. Although there is data for a number of cereal grains, including shelled maize, there is no information available on the cob as a whole.

THEORY

In this study, thermal diffusivity , α, is determined by graphical method and therefore an equation for thermal diffusivity is required .

The following assumptions were made for the convenience in theoretical analysis :

1. Cob is cylindrical,
2. There is no axial heat flow and
3. Thermal conductivity is independent of temperature.

The basic one-dimensional heat conduction equation in cylindrical polar coordinate with no heat generation is given by

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \qquad .....(1)$$

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where,
T = temperature at radius r, °C
t = time , sec
r = radius, m
α = thermal diffusivity of the material, m^2/sec

Defining the non-dimensional temperature ratio

$$\tau = \frac{T - T_A}{T_1 - T_A}$$

equation (1) becomes

$$\frac{\partial \tau}{\partial t} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \tau}{\partial r}\right) \quad \text{.........(2)}$$

Boundary conditions are,
t = 0, τ = 1; at surface, τ = 0

Consider a two-dimensional grid (t and r) and a typical point 'p' in that grid (Fig. 1).

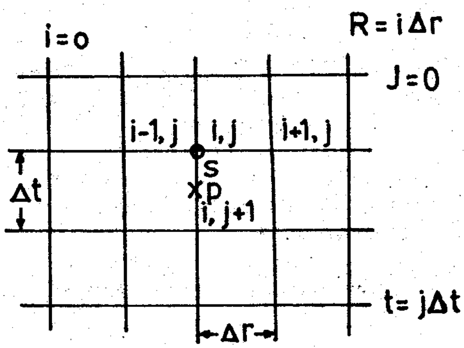

Fig. 1 Finite difference grid representation

The finite difference relationships for

$$\left.\frac{\partial \tau}{\partial t}\right|_P \quad \left.\frac{\partial \tau}{\partial r}\right|_P \quad \text{and} \quad \left.\frac{\partial^2 \tau}{\partial r^2}\right|_P$$

$$\left(= \left.\frac{\partial^2 T}{\partial r^2}\right|_S \quad \text{for small } \Delta t\right).$$

are

$$\left.\frac{\partial \tau}{\partial t}\right|_p = \frac{\tau_{i,j+1} - \tau_{i,j}}{\Delta t} \quad \text{.......(3)}$$

and

$$\left.\frac{\partial \tau}{\partial r}\right|_p = \frac{\tau_{i+1,j} - \tau_{i-1,j}}{2\Delta r} \quad \text{.......(4)}$$

$$\left.\frac{\partial^2 \tau}{\partial r^2}\right|_P = \frac{\tau_{i+1,j} - 2\tau_{i,j} + \tau_{i-1,j}}{(\Delta r)^2} \quad \text{.......(5)}$$

Substitution of equations (3), (4) and (5) in equation (2) gives

$$\frac{\tau_{i,j+1} - \tau_{i,j}}{\Delta t} = \alpha\left[\frac{\tau_{i+1,j} - 2\tau_{i,j} + \tau_{i-1,j}}{(\Delta r)^2} + \frac{1}{r_i}\frac{\tau_{i+1,j} - \tau_{i-1,j}}{2\Delta r}\right] \quad \text{.......(6)}$$

This may be arranged as follows :

$$\tau_{i,j+1} = \frac{\tau_{i+1,j} + \tau_{i-1,j}}{2} + \frac{\Delta r}{4 r_i}(\tau_{i+1,j} - \tau_{i-1,j}) \quad \text{.......(7)}$$

The Fourier number is

$$\frac{\alpha \Delta t}{(\Delta r)^2} = \frac{1}{2}$$

But for the center (r = 0)

$$\frac{1}{r}\frac{\delta \tau}{\delta r} \rightarrow \frac{0}{0},$$

which is indeterminate.

Applying L. Hospital's rule, we have

$$\frac{1}{r}\frac{\delta\tau}{\delta r}\Big|_{r\to 0} = \frac{\delta^2\tau}{\delta r^2}$$

So the heat conduction equation becomes

$$\frac{\delta\tau}{\delta t} = 2\alpha\frac{\delta^2\tau}{\delta r^2} \qquad ....(8)$$

Grid representation for center point is given in Fig. 2.

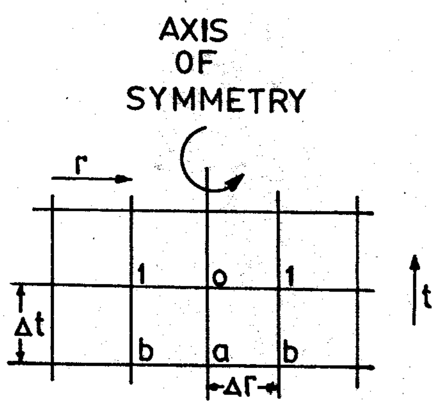

Fig. 2 Centre point grid representation

The finite difference relationships are :

$$\frac{\delta\tau}{\delta t}\Big|_p = \frac{\tau_a - \tau_o}{\Delta t} \qquad ....(9)$$

$$\frac{\delta^2\tau}{\delta r^2}\Big|_0 = \frac{\tau_1 - 2\tau_o + \tau_1}{(\Delta r)^2} \qquad ....(10)$$

$$\frac{\delta^2\tau}{\delta r^2}\Big|_a = \frac{\tau_b - 2\tau_a + \tau_b}{(\Delta r)^2} \qquad ....(11)$$

but

$$\frac{\delta^2\tau}{\delta r^2}\Big|_p = \left[\frac{\partial^2\tau}{\partial r^2}\Big|_o + \frac{\delta^2\tau}{\delta r^2}\Big|_a\right]/2 \qquad ....(12)$$

From equations (8), (9) and (12), the finite difference equation for center points can be obtained as :

$$\tau_a = \frac{\tau_1 + \tau_b}{2} \qquad ....(13)$$

From equations (7) and (13), the Fourier number, $(\alpha \Delta t)/(\Delta r)^2$ is equal to 1/2.

$$\text{i.e,} \frac{\alpha\,\Delta t}{(\Delta r)^2} = \frac{1}{2}$$

$$\text{or, } \Delta t = \frac{(\Delta r)^2}{2\alpha}$$

If n is the number of time steps, Δt, then

$$t = n\,\Delta t = \frac{n(\Delta r)^2}{2\alpha}$$

Therefore,

$$\ln(t) = \ln(n) + 2\ln(\Delta r) - \ln(2) - \ln(\alpha) \qquad ....(14)$$

In this study, τ vs ln(t) for experiment and τ vs ln(n) for theory was plotted. These two plots were compared and at any temperature ratio τ, the difference between ln(t) and ln(n) was defined as x.

So, equation (14) becomes:

$$2\ln(\Delta r) - \ln(2) - \ln(\alpha) = \ln(t) - \ln(n) = x$$

Therefore,

$$\alpha = \frac{(\Delta r)^2}{2\,e^x} \quad ....(15)$$

Equation (15) in conjunction with graphical values of x was used to calculate the thermal diffusivity.

MATERIALS AND METHODS

The cob used in this study was collected from the experimental farm of the University of Nairobi, Kenya. It had been stored in a refrigerator for 9 months. The cob from the lot were randomly selected. Its dimensions were measured by outside calipers. The cob was marked in three positions along its length; two positions at 40 mm inward from each end and one in the middle. Five readings were taken at each position and the average of those readings was considered as the diameter of the cob. The cob selected was 140 mm in length and 41 mm in diameter.

The cobs were placed inside a cylindrical box with about 300 ml of distilled water and then the box was kept on rolling for 48 hours. The conditioned cob was then kept in the atmosphere to allow the removal of gravitational water. Then four thermocouple were inserted axially in the cob at the center of the pith, at the periphery of the pith, at the root of the kernel, and within the germ part of the kernel. These four position were selected in order to determine the temperature throughout the cob (Fig. 3).

Three air-flow rates (1.14, 1.57, 2.33 m/s) were chosen to determine their effect on transfer of heat to the center. The heating temperature of the air has been maintained at 42.5° C ± 0.5° C during the experimental run.

Two experiments were conducted with each cob. In each of the experiments, one thermocouple was always kept for recording the air temperature. Experiment 1 consisted of heating with subsequent cooling and conducted with a view to establish the nature and rate of heat transfer in a maize cob. Experiment 2 was performed at lower temperatures to minimize the effect of drying and evaporation on temperature change.

Fig. 3 Maize cob showing thermocouple positions

Before starting any experiment, the air conditioning unit (AC unit), used in this experiment, was kept running for about half an hour to allow setting up of the air flow rate and air temperature. After half an hour of idle run of the AC unit, the instrumented cob, set previously by a piece of string, was suspended vertically in a duct of the AC unit. One reading was taken just before placing the cob into the unit. temperatures within the cob and the air stream were then logged at one minute intervals.

In experiment 1, the hot air of specified temperature (42.5° C $\pm$ 0.5° C) was allowed to pass through the duct to heat the cob. After heating, the cob was quickly placed in an oven previously set at a temperature of the air stream in AC unit.

The heaters of the AC unit were then switched off, but the blower fan was allowed to run with the same air flow. The temperature in the AC unit dropped to ambient air temperature within 20 minutes. It was checked with the thermometer placed in the laboratory. A further 10 minutes time was allowed before replacing the cob from the oven to the AC unit again.

In this way, the cooling run was done. The data logger, stopped at the time of removing the cob from the AC unit, was re-started. The temperature in the center of the maize cob was supposed to reach the temperature of the ambient air stream during cooling run, but it did not do so after several hours.

In experiment 2, the cob was cooled in a freezer for 24 hours before starting the experiment. The cob was then taken out of the freezer and suspended in respective section of the AC unit which was kept running with 1.57 m/s ambient air flow for half an hour.

The outputs of experiment 2 were recorded in a similar way followed in experiment 1. From these data temperature ratio and ln(t) were calculated and then temperature-time, experimental τ versus ln(t), and theoretical τ versus ln(n) curves were plotted. Thermal diffusivities, α, for all runs were then calculated using these two sets of experimental and theoretical curves. To obtain corresponding values of n and t, the experimental and the theoretical curves were superimposed to intersect at $\tau = 0.5$. Finally, time $t = (n\Delta t) = [n(\Delta r)^2/2\alpha]$ for theoretical τs were calculated and experimental and theoretical τ versus ln(t) were plotted in the same figure to check the agreement.

RESULTS AND DISCUSSION

Three temperature versus time graphs incorporating heating and cooling for experiment 1 are shown in Fig. 4(a), 4(b) and 4(c). In each of them the region of the germ (thermocouple at R = R3) shows a rapid temperature response which leads to the assumption that the coefficient of conduction is lower in a layer between the germ and the root of the kernels than that in the core where the temperatures are fairly uniform.

During heating, evaporation effect resists the temperature of the cob to reach the air temperature which is evident from the gap between air temperature and the temperature inside the maize cob towards the end of heating. It is further confirmed from the ambient heating curve, Fig. 4(d) where the gap is a minimum due to lower rates of evaporation. On the other hand, cooling indicates very negligible moisture movement [Fig. 4(a), 4(b) and 4(c)].

The temperature of the maize cob reaches the air temperature within an hour. Further analysis of these curves indicates that a considerable steep slope in these curves occurs in the time range of 20 minutes.

The temperature ratio vs. ln(t) curves for heating of the cob at three velocities stated earlier are shown in Fig. 5. These curves suggest that the effect of air velocity is very small in increasing the rate of change of center temperature. These also indicate that the air velocity is not critical in determining the heat transfer rate to or from a maize cob over this velocity range.

Experimental and theoretical temperature ratio vs. ln(t) are shown in Fig. 6 and Fig. 7. The experimental curves are in very good agreement with the theoretical curves. On the other hand, the curves for heating show a lower slope for experimental than theoretical results, and the temperature ratio never reaches zero, which again is due to evaporation.

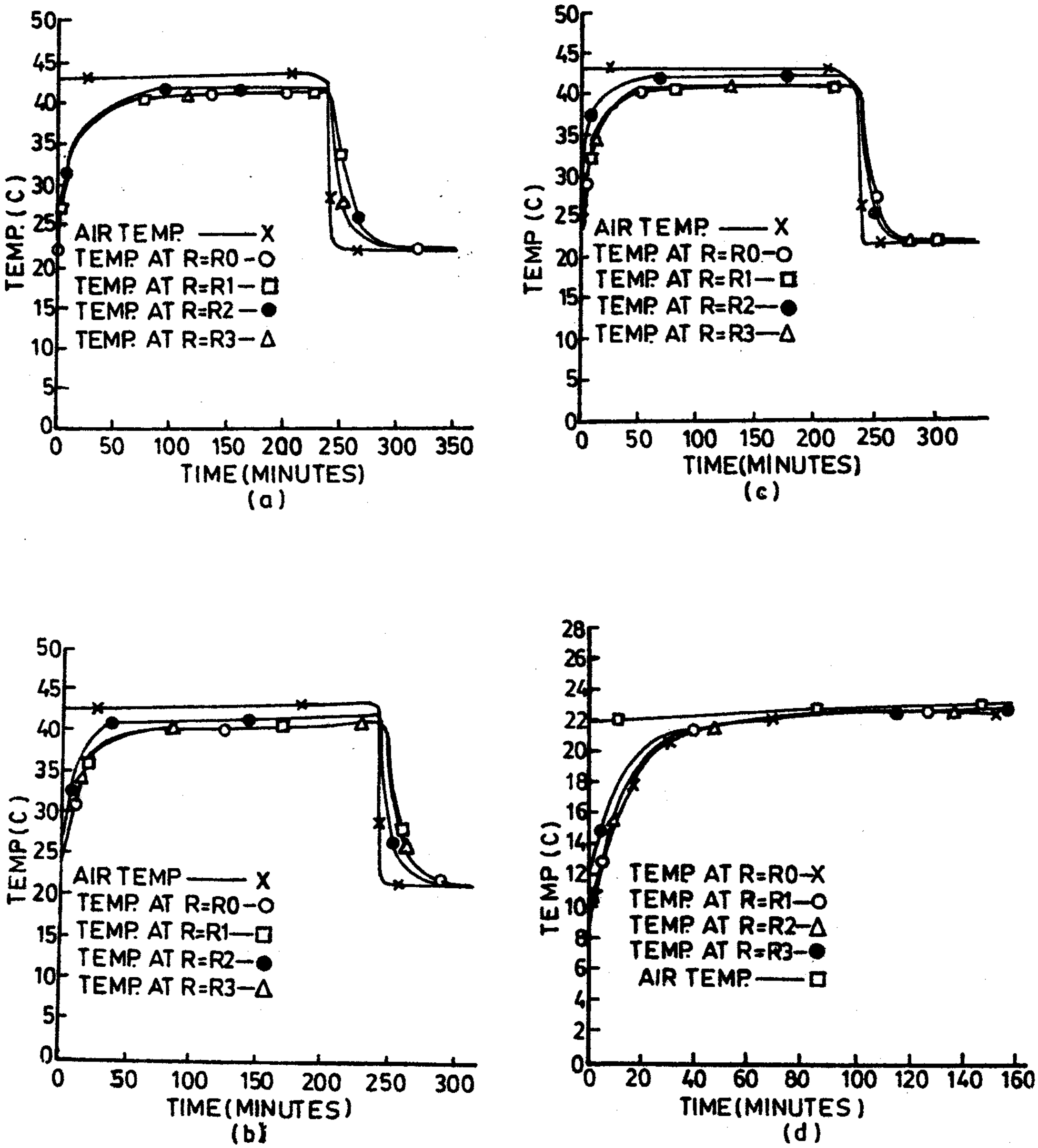

Fig. 4 Temperature-time curves for heating and cooling of the maize cob at an air flow rate of (a) 1.14 m/s, and (b) 1.57 m/s, (c) 2.33 m/s, and (d) ambient heating at 1.57 m/s.

Thermal diffusivities were calculated by matching experimental and theoretical curves at $\tau = 0.5$ and these values range from 0.0098×10^{-5} m^2/s to 0.0128×10^{-5} m^2/s. It is also noticed that if the thermal diffusivity were calculated for the values of τ other than 0.5, different results could be obtained.

CONCLUSION

The increase in air velocity slightly increases the rate of change of center temperature, but the effect is small. This suggests that the resistance to heat transfer is largely within the maize cob.

Temperatures are largely uniform during heating or cooling in the central region but the germ part of the kernel shows a rapid temperature response. These results suggest that the resistance to heat transfer is in a layer between the germ and the root of the kernels. Ambient heating for the cob shows a similar effect.

The thermal diffusivities found in this study fall in the range from 0.007×10^{-5} to 0.0128×10^{-5} m^2/s.

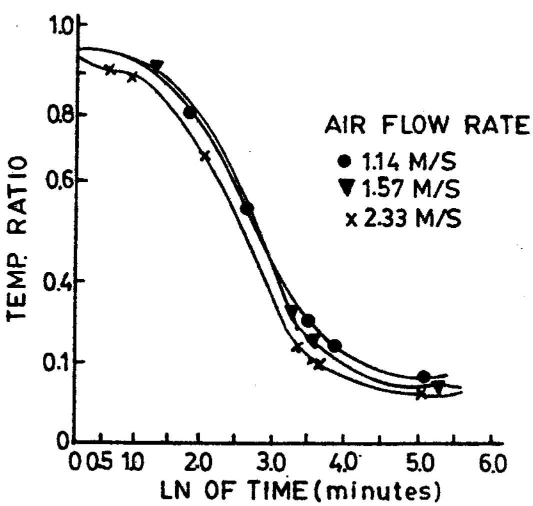

Fig. 5 Temperature ratio curves τ vs ln(t) in heating of cob for 3 air flow rates

The shape of the cooling curves are in better agreement with theory than the heating curves during the final stages ($\tau \rightarrow 0$). This might be due to evaporative cooling.

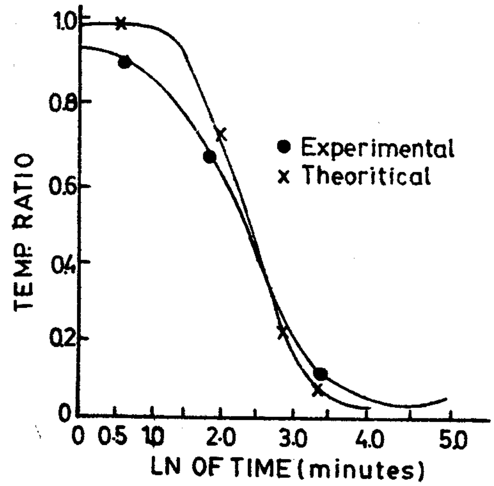

Fig. 6 Theoretical and practical τ vs ln(t) curves in ambient heating of cob at an air flow rate of 1.57 m/s

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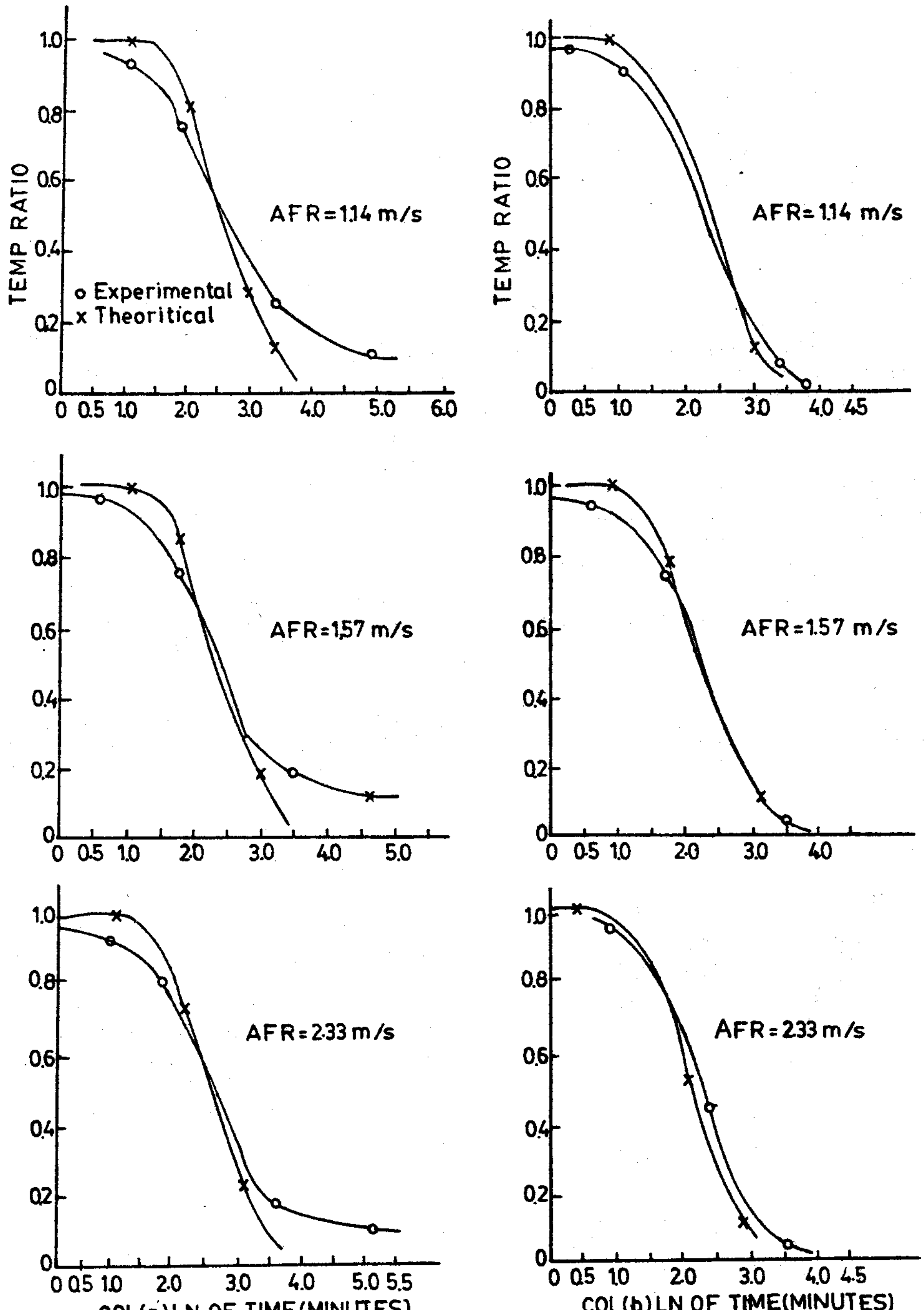

Fig. 7 Theoretical and practical τ vs ln (t) curves at different air flow rates
Column (a) heating, column (b) cooling